Individual Analytical Analysis

CWC Generator
Shaft Analysis

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Disclaimer
This analysis was going to talk about how the Arduino calculated measurements off a Dynamometer like voltage, current, RPM, and torque. However, these values are calculated by simple conversions or a small equation. It does not provide much of an analysis. So, this analysis will now talk about the shaft inside a generator and what it will experience. Thank you for understanding and taking the time to read this.

Introduction
The Northern Arizona University (NAU) Collegiate Wind Competition (CWC) team typically buy motors to use as a permanent magnetic synchronous generator (PMSG) or design a PMSG to their specific needs for their wind turbine in the Department of Energy (DOE) CWC. Since the NAU CWC team buy PMSGs, they will need to design around it to ensure it is used efficiently. Also, these PMSGs can be back ordered which can cause delay for the NAU CWC team to design their wind turbine. This project will aim to solve these inconveniences and problems by making a PMSG that can adjust its parameters, like voltage, current, torque, speed, and resistive torque.

To start achieving this goal is by analyzing how the generator connected to a wind turbine will be affected by the aerodynamic forces. This paper will analyze the shaft from the PMSG that will experience aerodynamic forces, reaction forces, moments, torques, etc. This analysis will also help the design process of the PMSG so it can withstand worst case scenarios that it might experience in the DOE CWC.

Analysis
This section of the paper will talk about how this analysis was approached and discuss its findings. This includes assumptions, equations, constraints, schematics, and results. With these considerations, this will determine how to design the PMSG shaft. In addition, this analysis uses a programming software, Matrix Laboratory (MatLab), to compute the following analysis, which can be seen in Code B.1 and Code B.2.

Assumptions
This section will talk about assumptions made to reach a result of this analysis along with their reasoning why they are needed.

Starting off, simple assumptions are neglecting the weight of the shaft, the weight of the blades and hub being 2 lbs (0.9 kg) total, and the only reaction forces are acted on the bearings. Neglecting the weight of the shaft was considered since the weight of the shaft will provide negligible deformation due to its small size. The weight of the blades and hub are not neglected since it was discussed with the client of this project, Professor David Willy, to assign the weight as 2 lbs. This weight is valid because the airfoils on wind turbines are made to be light, and the hub is a small piece. In addition, this value is an overestimation of the weight since we want to analyze the shaft in worst-case scenarios. To end this segment, bearings are the main supports of this shaft since it rotates with the shaft, and it will have higher reaction forces compared to forces acting on the housing walls for the shaft for example. These bearings specifically are a thrust and a journal bearing.

Moving onto the significant assumptions, the only aerodynamic force acting on this part is thrust force. The thrust force is a translation from lift, where it helps the generator provide power. This allows a simpler calculation of estimating bending moments along the shaft. Also, this thrust force will be calculated by assuming that the generator will be operating at Betz Limit, which will be applied in the thrust equation shown later. Additionally, moments acting on the bearing will be ignored as there will be too many unknowns to continue with this shaft analysis. Ignoring magnetic forces will be considered since there are no values to calculate these forces now. The shaft will be aluminum since many generators' shafts are made of aluminum [1]. Lastly, this analysis will be considered at sea level with a temperature of 0 ℃ where the shaft is in equilibrium, except axial rotation. Table 1 will be used to talk about which assumptions were used during the explanation of equations and calculations.

Table 1 - Assumptions listed with a corresponding number

Number	Assumption
1	Neglect shaft weight
2	Weight of blades and hub is 2 lbs
3	Bearing supports (Journal and Thrust)
4	Only aerodynamic force is thrust
5	Generator is perfectly efficient (Betz Limit)
6	Neglect moments on bearings
7	Ignore magnetic forces
8	Conservative
9	Shaft is aluminum
10	At sea level with a temperature of 0 ℃
11	Shaft in equilibrium, except axial rotation

Equations

Most equations listed below will be based on assumptions from Table 1 but will show the derivation of it, if applicable.

Equations 1, 2, and 3 are the baseline of this analysis which will help find the parameters for determining the diameter of the shaft, like shear force and bending moment.

$$W = mg \tag{1}$$

W – Weight [N]
m – Mass [kg]
g – Gravity [m/s^2]

$$\Sigma F = 0 \tag{2}$$

F – Force acting on the part [N]

$$\Sigma M = 0 \tag{3}$$

M – Moment acting on the part [N-mm]

Both Equations 2 and 3 are both set to 0 because of Assumption 11.

Moving on, we know this shaft will be rotating axially, which means it will provide a torque. This can be calculated by Equation 4.

$$T = \frac{P}{\omega} \tag{4}$$

T – Generator torque [N-mm]
P – Generator power [W]
ω – Shaft Angular velocity [rad/s]

$$\omega = Kv * V \tag{5}$$

ω – Shaft angular velocity [RPM]
Kv – Voltage constant [RPM/V]
V – Generator voltage output [V]

Equation 4 calculates torque with P and ω. P can be found in Figure A.1, but ω will not be given. However, ω can be found using Equation 5, since Kv rating and voltage is given in Figure A.1.

$$F_T = 2\pi R^2 \rho U^2 a(1-a) \tag{6}$$

F_T – Thrust force [N]
R – Blade radius [m]
ρ – Air density [kg/m^3]
U – Air velocity [m/s]
a – Inference factor

$$F_T = \frac{4\pi R^2 \rho U^2}{9} \tag{6.1}$$

F_T – Thrust force [N]
R – Blade radius [m]
ρ – Air density [kg/m^3]
U – Air velocity [m/s]

Equation 6 applies Assumption 4 and 5 by getting the general equation for thrust force on a wind turbine from [2, eq. (10.42)] and Betz was able to show that the theoretical efficiency is maximized when a=1/3 [2]. So, Equation 6 can be rewritten as Equation 6.1 to find the thrust force.

Next, the equations to help find a good starting point on the diameter of the shaft is using Equations 7, 8, and 9.

$$S_e = k_a k_b k_c k_d k_e k_f S'_e \tag{7}$$

S_e – Endurance limit [MPa]
k_a – Surface condition modification factor
k_b – Size modification factor
k_c – Load modification factor
k_d – Temperature modification factor
k_e – Reliability modification factor
k_f - Miscellaneous-effects modification factor
S'_e- Test specimen endurance limit [MPa]

$$S'_e = 0.5S_{ut} \tag{8}$$

S'_e- Test specimen endurance limit [MPa]
S_{ut} – Ultimate strength [MPa]

$$k_a = aS_{ut}^b \tag{9}$$

k_a – Surface condition modification factor
S_{ut} – Ultimate strength [MPa]
a – Factor for surface finish
b – Exponent for surface finish

Equation 8 comes from [3, eq. (6-8)] and has many constraints for determining the endurance limit, S'_e. Since Assumption 9 is applied, the ultimate strength, S_{ut}, can be found in [3, Tab. A-22] for aluminum. With only three aluminum ultimate strengths being listed, the highest is 593 MPa which will only satisfy the constraint of $S_{ut} \leq 1400\ MPa$ [3]. Therefore, this will only allow the analysis to use Equation 8 for the test specimen endurance limit, S'$_e$. For the modification factors in Equation 7, some will be considered since some of these factors rely on values that are pre-determined, like diameter and temperature. Therefore, the values k_a, k_b, and k_c will be the main modification factors that are going to be calculated or guessed. Otherwise, the other modification factors will be 1.

Lastly, we have equations that will calculate the diameter of the shaft, diameter of the thrust lip, and shoulder fillet radius based on the previous equations.

$$d = \{\frac{16n}{\pi}(\frac{2K_f M_a}{S_e} + \frac{1}{S_{ut}}[3(K_{fs}T_m)^2]^{\frac{1}{2}})\}^{\frac{1}{3}} \tag{10}$$

d – Diameter of shaft [mm]
n – Safety factor
K_f – Fatigue stress concentration factor
M_a – Alternating moment [N-mm]
S_e – Endurance limit [MPa]
S_{ut} – Ultimate strength [MPa]
K_{fs} – Fatigue stress concentration factor
T_m – Mid-range torque [N-mm]

$$D = 1.2d \tag{11}$$

D – Thrust lip diameter [mm]
d – Shaft diameter [mm]

$$r = d/10 \tag{12}$$

r – Shoulder fillet radius
d – Shaft diameter

Equations 10 and 11 will help determine the diameter of the shaft as well as the thrust lip diameter for the thrust bearing that it will be leaning against on as thrust force is being applied. This shaft will experience constant torque from the generator and constant bending from the thrust force, so midrange bending moment and alternating torque will not occur. This helps deriving Equation 10 from [3, eq. (7-8)]. After finding the shaft diameter, the thrust lip diameter can be found with Equation 11. A typical D/d ratio is 1.2 for a curved filet [3]. Lastly, a well-round shoulder fillet will be applied to the shaft with Equation 12 [3].

Physical Modeling

Material

Considering Assumption 8 and 9, this shaft material will be aluminum with a low ultimate strength from [3, Tab. A-22] to calculate a conservative diameter of the shaft and thrust lip. In this case, the aluminum will be 2011-T6 Aluminum with a density of 2830 kg/m^3 [4] and S_u will be 324 MPa [3, Tab. A-22].

Dimensions

Although the diameter of the shaft and thrust lip is not yet determined, it can still be calculated with a 15 mm length of the shaft. A 15 mm length of the shaft was chosen since typical generators tend to be 10 mm for a compact design [1]. However, a thrust lip is being introduced to the design which is why the length is slightly longer than typical compact generators. Also, it is typical to keep the shaft short to reduce bending moment along it [3]. The blade diameter of the turbine will be 45 cm (22.5 cm in radius) from Figure A.1. Lastly, Professor Willy guessed the height of the centerline to the thrust force to be 7 cm. This was introduced to the dimensions since the thrust force will be creating a bending moment along the shaft due to the turbine wanting to move forward in a real-world scenario. In addition, Professor Willy mentioned this to be a good, overestimated value for this analysis.

Generator Outputs

The generator power, Kv rating, and maximum voltage output are 10 kW, 150, and 48 V, respectively. These values can be found in Figure A.1

Schematics

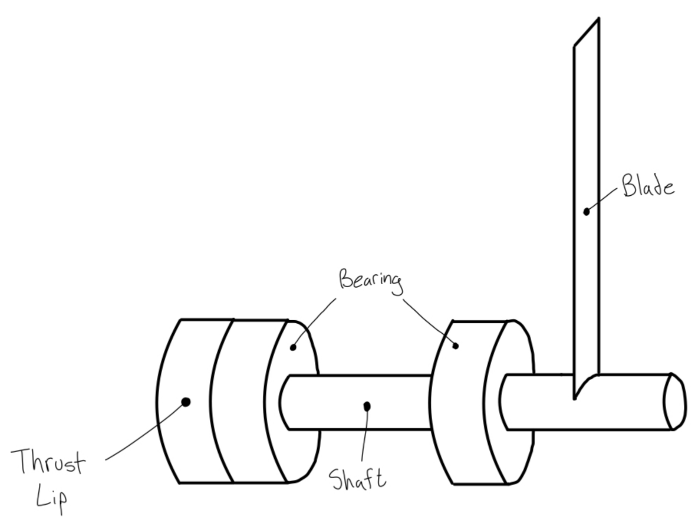

Figure 1 – Schematic of part being analyzed

In Figure 1, the part is displayed where it includes the shaft, bearings, and blade that will be experiencing the thrust force.

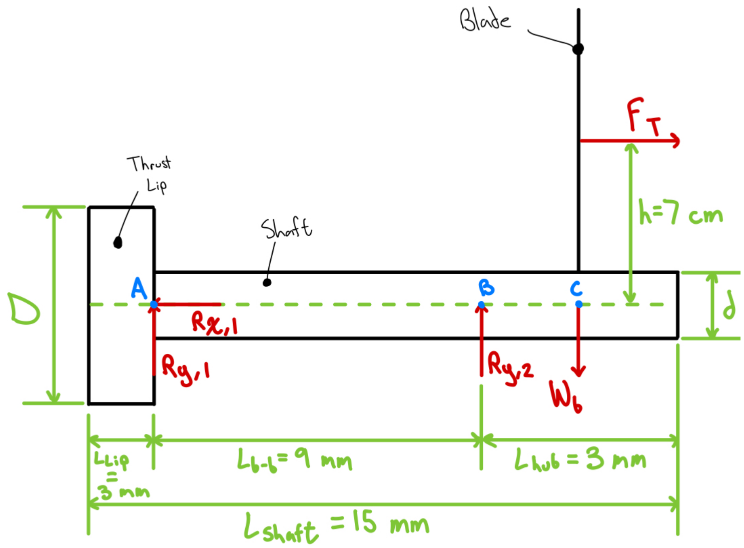

Figure 2 – FBD of shaft with blade and hub

In Figure 2, it converted Figure 1 into a FBD to analyze how the shaft will react with the forces applied to it.

Calculations

Finding the blade and hub weight force, and torque applied from the generator will be calculated first due to their simple equations. The mass of the blade and hub are known, which means the weight can easily be found by considering Assumption 10. This means gravity, g, will be 9.81 m/s^2. Equation 1 can be applied to find the weight of the blade and hub to be 8.90 N. For torque, it was mentioned previously that angular velocity needs to be found first before calculating torque. Based on Figure A.1, generator power, Kv, and V are 10 kW, 150 and 48 V, respectively. Equation 5 can be applied to find angular velocity of the shaft, which then can be applied to Equation 4 to find the shaft torque to be 13300 N-mm.

Next, the thrust force is also needed to start calculating reaction forces of the bearings. After applying Equation 6.1 with an air density of 1.293 kg/m^3 [2] and air speed of 13 m/s, the thrust force was found to be 15.4 N. In addition, there are only 2 x direction forces. By inspection, it is clear the axial force on the thrust bearing is also 15.4 N to satisfy equilibrium on the part in the x direction. Before moving forward, the air speed was discussed with the client and recommended 13 m/s due to CWC guidelines.

Next, the y direction reaction forces can be found by applying Equation 3 for the z axis at either point A or B. Assuming moments will be evaluated at point A, the equation is

$$+\left[R_{y,2}(L_{b-b})\right] - \left[W_b\left(L_{b-b} + \frac{L_{hub}}{2}\right)\right] - (F_T h) = 0$$

$R_{y,2}$ – Second bearing reaction force in y direction [N]
$L_{b\text{-}b}$ – Length of bearing-to-bearing [mm]
W_b – Weight of blades and hub [N]

L_{hub} – Length of hub [mm]
F_T – Thrust force [N]
h – Centerline of shaft to thrust force distance [mm]

Solving for the second reaction bearing force, $R_{y,2}$, it is found to be 190.6 N. This value can then be applied by summing forces in the y direction with

$$+R_{y,1} + R_{y,2} - W_b = 0$$

$R_{y,1}$ – First bearing reaction force in y direction [N]
$R_{y,2}$ – Second bearing reaction force in y direction [N]
W_b – Weight of blades and hub [N]

This will solve for the first reaction bearing force, $R_{y,1}$, which is -181.7 N. In this case, the value is negative meaning that the force is acting down.

All forces and torque have been calculated with Figure 2's FBD. Now, a torque, shear force, and bending moment diagram can be made.

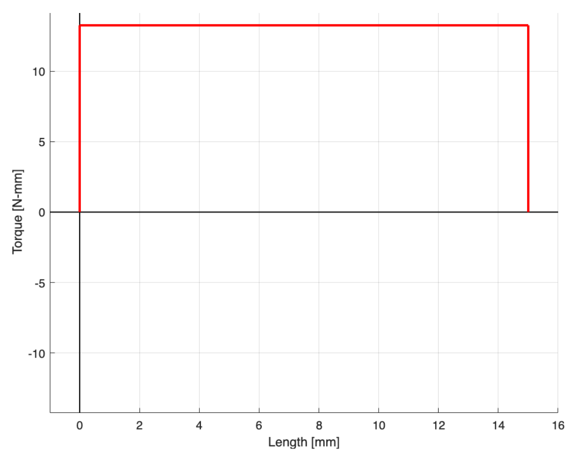

Figure 3 – Torque Diagram

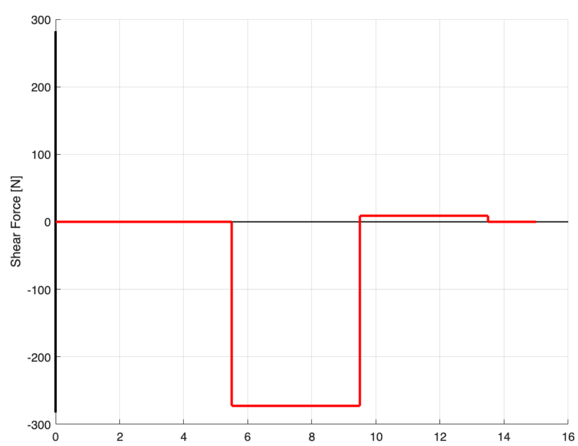

Figure 4 – Shear force diagram

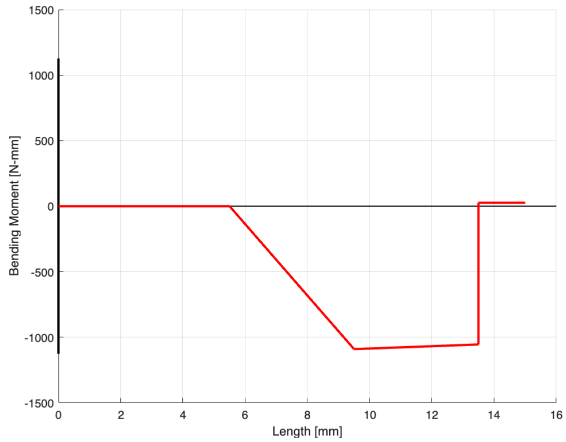

Figure 5 – Bending moment diagram

Figures 3 shows the torque being constant along the shaft which makes sense since this shaft will be constantly rotating. Figure 4 shows the shear force effects, and it shows that the most shear force is being applied between the bearings with minimal shear after the second bearing. Lastly, Figure 5 shows the bending moment acting along the shaft. Most of the bending will occur near the second bearing. One thing that can be seen in Figure 5 is that it does not end in zero which means there is another moment acting on the shaft since we want this part to be in equilibrium. However, this moment is negligible. If it were considered, it would mean a slight increase in bending moment near the second bearing.

Next step to finding diameters is finding the endurance limit. As mentioned before, k_a, k_b, and k_c will be the ones considered meaning the other modification factors will be set to 1 since they require pre-defined properties that is not given in this analysis. k_b, and k_c are 0.9 and 1, respectively. K_b is a guess to keep the analysis conservative on the shaft diameter [3] and k_c is set to 1 due to the shaft experiencing bending based on [3, eq. (6-26)]. For finding k_a, a, b, and S_u are 2.7, 4.51, and 324 MPa, respectively. a and b were determined by [3, Tab. 6-2] since this shaft will be machined, and S_u was found by [3, Tab. A-22]. Applying Equation 8 and 9 to 7, S_e is 85.1 MPa. Now applying Equation 10 with a safety factor, n, of 3 and a well-rounded shoulder fillet stress concentration factors, K_t and K_{ts}, 1.7 and 1.5, respectively. Diameter of the shaft is calculated to be 4.63 mm. Now applying Equation 11, the diameter of the thrust lip is 5.56 mm. Lastly, applying equation 12, the radius for the shoulder fillet will be 0.463 mm.

Results

After performing calculations with the equations listed above, the resulting diameter of the shaft and thrust lip were 4.63 mm and 5.56 mm. For finding these diameters, safety factor is the main parameter that needs to be considered. Since there were many assumptions taken place in this analysis, there were many deformation effects not considered on the shaft. So, a safety factor of 3 was used to substitute the effects that were not considered in this analysis. In addition, this project is not going to harm the public/users, so a safety factor of 3 is reasonable for at least considering both safety and unconsidered effects on the shaft.

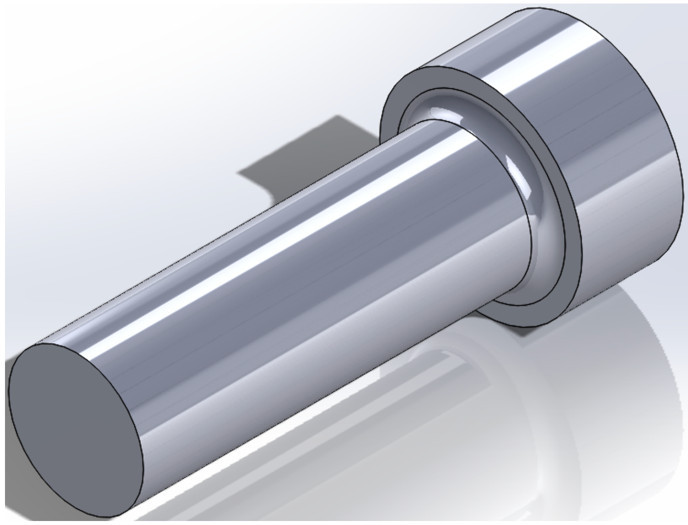

Figure 6 – 3D model of shaft moving forward in design process

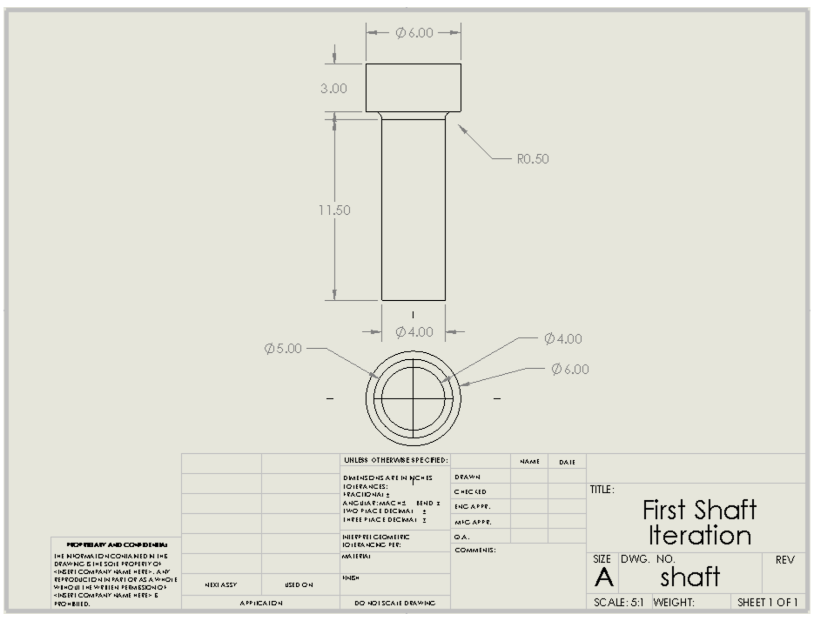

Figure 7 – Shaft drawing with dimensions

Conclusion

In conclusion, many equations were applied to find the diameters of the shaft and thrust lip that will withstand the forces acting on the shaft. With these results, it can be applied to the project's design process. This analysis used a safety factor of 3 and was conservative throughout the whole process. So, it is safe to say that the diameter of the shaft, diameter of the thrust lip, and shoulder fillet radius will be 4 mm and 6 mm, and 0.5 mm, respectively. In Figure 6 and 7, the computer-aided design of the shaft is displayed with the dimensions that will be moving forward to the design of the generator. Moving forward, bearing selection and stator design will be the next consideration of the design process so the generator can work properly and efficiently for the NAU CWC team.

Appendix

Part A: Figures

House Of Quality			Technical Specifications (How)											
			Maximum 48 Volts	45 cm Roter Diameter of Turbine	Low Total Resistant Torque	Low Kv Rating	Magnetic Flux	Turbine Power	Generator Power	Number of Coils	Tip Speed Ratio	Diemeter of coil	Cut Out Speed	Cut In Speed
	Customer Requirements (What)	Importance	1	2	3	4	5	6	7	8	9	10	11	12
1	Low Voltage	96	9	3	9	9	9	9	9	9	3	9	9	9
2	Small Size	60	3	9	3	1	3	9	3	9	9	9	1	1
3	High Power Generation	100	9	1	9	9	9	9	9	9	9	9	9	9
4	Under Budget	28	1	3	1	1	1	3	3	3	3	3	3	3
5	Ability to change easy	60	1	3	3	9	3	3	9	9	9	9	1	1
6	Up to design standards for CWC	102	9	9	9	9	9	9	9	3	9	9	9	9
7	3 phase AC	34	3	3	3	3	3	1	3	9	1	3	1	1
		Target	48 V	45 cm	2 N-m	150	1-3 T	100 kW	0.5-10 kW	6 coils	7 to 8	0.321-0.644 mm	25 m/s	3 m/s
		Importance	35	31	37	41	37	43	45	51	43	51	33	33

Figure A.1 - Quality Function Deployment (QFD)

Part B: MatLab Code

Code B.1 - Shaft Analysis

```
V = 48; % V
P = 10; % kW
Kv = 150; % Kv Rating
D_b = 45; % cm
m_b = 2; % lb (rough approx from Willy)
h = 7; % cm Distance for aerodynamic forces (thrust)
L_lip = 3; % mm (subject to change)
L_b_b = 9; % mm (Length from end of bearing to other end of bearing)
L_hub = 3; % mm (Length where connection from blade will occur)
rho_al = 2830; % kg/m^3
Su = 324; % MPa (2011 Aluminum in Table A-22)
Kt = 1.7; % Assuming we're using a rounded filet in Table 7-1
Kts = 1.5; % Assuming we're using a rounded filet in Table 7-1
Kf = Kt; % Quick Conservative test
Kfs = Kts; % Quick Conservative test
a = 2.7; % Table 6-2
b = -0.265; % Table 6-2
Ka = a * Su^b; % Eq. 6-19
Kb = 0.9; % Guess
Kc = 1;
Kd = Kc;
Ke = Kd;
n = 3;
w = 5; % mm
U = 13; % m/s
rho_air = 1.293; % kg/m^3
g = 9.81; % m/s^2
omega = ( Kv * V ) * 2 * pi / 60; % rad/s
T = P * 10^3 / omega; % N-mm
D_b = D_b / 100; % m
R_blade = D_b / 2; % m
m_b = m_b * 0.45359237; % kg
h = h * 10; % mm
L_total = L_lip + L_b_b + L_hub; % mm
FT = get_thrust_force( R_blade, rho_air, U ); % Assuming Betz Limit (a=1/3)
W_blade = m_b * g;
Rx1 = FT;
Ry2 = ( L_b_b - w )^( -1 ) * ( W_blade * ( L_b_b - w / 2 - L_hub / 2 ) + FT * h );
Ry1 = W_blade - Ry2;
d1 = L_lip + w / 2; % mm
d2 = L_b_b - w;
d3 = w /2 + L_hub / 2; % mm
d4 = L_hub / 2; % mm
d = [ d1, d2, d3, d4 ];
Fv = [ 0, Ry1, Ry2, -W_blade ];
```

```
Fv_max = max( abs( Fv ) );
figure( "Name", "Torque Diagram")
hold on
% x and y axis line
plot( [ -1, L_total + 1 ], [ 0, 0 ], "black", "LineWidth", 1 )
plot( [ 0, 0 ], [ -T - 1, T + 1 ], "black", "LineWidth", 1 )
% Torque curve
plot( [ 0, 0 ], [ 0, T ], "r", "LineWidth", 2 )
plot( [ 0, L_total ], [ T, T ], "r", "LineWidth", 2 )
plot( [ L_total, L_total ], [ T, 0 ], "r", "LineWidth", 2 )
% Labeling
xlabel( "Length [mm]" )
ylabel( "Torque [N-mm]" )
xlim( [ -1, L_total + 1 ] )
ylim( [ -T - 1, T + 1 ] )
grid on
hold off
figure( "Name", "Shear Force Diagram" ); hold on
plot( [ 0, L_total + 1 ], [ 0, 0 ], "black", "LineWidth", 1 )
plot( [ 0, 0 ], [ -Fv_max - 1, Fv_max + 1 ], "black", "LineWidth", 2 )
figure( "Name", "Bending Moment Diagram" ); hold on
plot( [ 0, L_total + 1 ], [ 0, 0 ], "black", "LineWidth", 1 )
plot( [ 0, 0 ], [ -Fv_max * d2 - 1, Fv_max * d2 + 1 ], "black", "LineWidth", 2 )
L_start = 0;
L_end = 0;
Fv_total = 0;
M = 0;
for i = 2 : length( d ) + 1
    L_end = L_end + d( i - 1 );
    Fv_total = Fv_total + Fv( i - 1 );
    x = linspace( L_start, L_end, 1000 );
    M = M( end ) + Fv_total * ( x - L_start );
    figure( findobj('Name', 'Shear Force Diagram') )
    plot( [ L_start, L_end ], [ Fv_total, Fv_total ], "r", "LineWidth", 2 )
    if i < length( d ) + 1
        plot( [ L_end, L_end ] , [ Fv_total, Fv_total + Fv( i ) ], "r", "LineWidth", 2 )
    end
    figure( findobj('Name', 'Bending Moment Diagram') )
    plot( x, M, "r", "LineWidth", 2 )
    if i == length( d )
        Ma = -M( end );
        plot( [ L_end, L_end ], [ M( end ), M( end ) + FT * h ], "r", "LineWidth", 2 )
        M = M( end ) + FT * h;
    end
    L_start = L_end;
end
figure( findobj('Name', 'Shear Force Diagram') )
xlabel( "Length [mm]" )
ylabel( "Shear Force [N]" )
grid on
figure( findobj('Name', 'Bending Moment Diagram') )
xlabel( "Length [mm]" )
ylabel( "Bending Moment [N-mm]" )
grid on
Se_prime = 0.5 * Su; % MPa
Se = Ka * Kb * Kc * Kd * Ke * Se_prime; % MPa
d_shaft = ( 16 * n / pi * ( 2 * Kf * Ma / Se + ( 3 * ( Kfs * T )^2 )^( 1 / 2 ) / Su )^( 1 / 2 ) )^( 1 / 3 );
fprintf( "Shaft Diameter: %.2f mm\n", d_shaft )
D_over_d_ratio = 1.2;
D_lip = D_over_d_ratio * d_shaft;
fprintf( "Thrust Lip Diameter: %.2f mm\nThis value is assumed to be\nconservative, so feel free\nto choose a diameter thats\nlower than the value 
displayed", D_lip )
```

Code B.2 - Thrust Force Calculation

```
function FT = get_thrust_force( R, rho, V, varargin )
    % Inputs
        % R - Radius of blade, in m
        % rho - Density of liquid, in kg/m^3
        % V - Velocity of liquid speed, in m/s
        % varargin - If you want to change inference factor, a, then put
        % one extra argument in function
    % Output
        % FT - Thrust force acting on wind turbine, in N
    % Description
        % get_thrust_force.m calculates thrust force based on Equation
        % 10.42 found in the Fluid Mechanics book. It does this by assuming
        % Betz Limit. It can have the option to not be based on Betz Limit
        % if you add one extra argument for the inference factor
    a = 1 / 3; % Assuming Betz Limit
    % If changing inference factor, then change it
    if nargin == 4
        a = varargin{ 1 };
    % If too many arguments, keep inference factor the same.
    elseif nargin > 4
        disp( "Too many arguments. Using a = 1/3" )
    end
    % Calculate thrust force
    FT = 2 * pi * R^2 * rho * V^2 * a * ( 1 - a );
end
```

References

[1] "5010 brushless motor 360kv 2-6s high torque motor for 14-18" props," SpeedyFPV, https://speedyfpv.com/products/5010-brushless-motor-360kv-2-6s-high-torque-motor-for-14-18-props?srsltid=AfmBOooF8MI98pmLNLayj2T48VVjJQpWbU3K3U70aneZYgSFboUYEuYH&variant=42912725860566 (accessed Apr. 24, 2025).

[2] J. W. Mitchell, R. W. Fox, and A. T. McDonald, *Fox and McDonald's Introduction to Fluid Mechanics*. Hoboken NJ: John Wiley & Sons, Inc, 2020.

[3] J. K. Nisbett and R. G. Budynas, *Shigley's Mechanical Engineering Design*. New York, NY: McGraw-Hill, 2024.

[4] "The Online Materials Information Resource," MatWeb, https://www.matweb.com/search/datasheet.aspx?matguid=66a81429bea54053bbdc39cfce0f2407&ckck=1 (accessed Apr. 24, 2025).